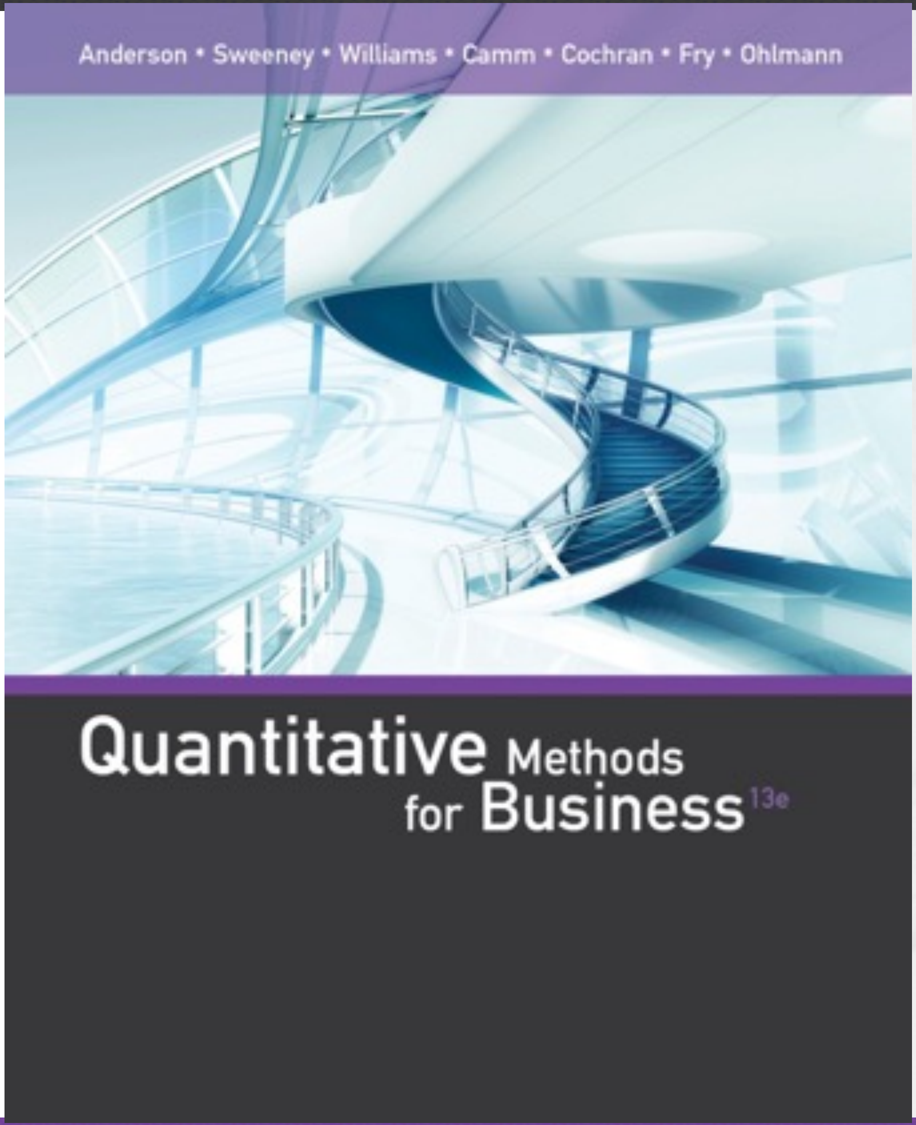
Slides by

John Loucks

St. Edward's University



Chapter 10, Part A

- Supply Chain M
  - Transportatic
  - Transshipme

FIGURE A.16 EXCEL WORKSHEET USED TO CALCULATE TOTAL SHIPPING COSTS FOR THE FOSTER GENERATORS TRANSPORTATION PROBLEM

WEB file
FosterGenerators

	A	B	C	D	E	F	G	H
1	**Foster Generators**							
2								
3			**Destination**					
4	**Origin**	Boston	Chicago	St. Louis	Lexington	**Supply**		
5	Cleveland	3	2	7	6	5000		
6	Bedford	7	5	2	3	6000		
7	York	2	5	4	5	2500		
8	**Demand**	6000	4000	2000	1500			
9								
10								
11	**Model**							
12								
13		**Min Cost**						
14								
15			**Destination**					
16	**Origin**	Boston	Chicago	St. Louis	Lexington	**Total**		
17	Cleveland	3500	1500	0	0	5000	<=	5000
18	Bedford	0	2500	2000	1500	6000	<=	6000
19	York	2500	0	0	0	2500	<=	2500
20	**Total**	6000	4000	2000	1500			
21		=	=	=	=			
22		6000	4000	2000	1500			

Figure A.16 displays an Excel worksheet for the Foster Generators Problem that appears in Chapter 10. This problem involves the transportation of a product from three plants (Cleveland, Bedford, and York) to four distribution centers (Boston, Chicago, St. Louis, and Lexington). The costs for each unit shipped from each plant to each distribution center are shown in cells B5:E7, and the values in cells B17:E19 are the number of units shipped from each plant to each distribution center. Cell C13 will contain the total transportation cost corresponding to the transportation cost values in cells B5:E7 and the values of the number of units shipped in cells B17:E19.

The following steps show how to use the SUMPRODUCT function to compute the total transportation cost for Foster Generators.

Step 1. Select **cell C13**
Step 2. Click *fx* on the formula bar
Step 3. When the **Insert Function** dialog box appears:
Select **Math & Trig** in the **Or select a category** box
Select **SUMPRODUCT** in the **Select a function** box (as shown in Figure A.15)
Click **OK**
Step 4. When the **Function Arguments** box appears (see Figure A.17):
Enter *B5:E7* in the **Array1** box
Enter *B17:E19* in the **Array2** box
Click **OK**

The worksheet then appears as shown in Figure A.18. The value of the total transportation cost in cell C13 is 39500, or $39,500.

Transportation, Transshipment, and Assignment Problems

- A network model is one which can be represented by a set of nodes, a set of arcs, and functions (e.g. costs, supplies, demands, etc.) associated with the arcs and/or nodes.
- Transportation and transshipment problems of this chapter as well as the PERT/CPM problems (in Chapter 13) are all examples of network problems.

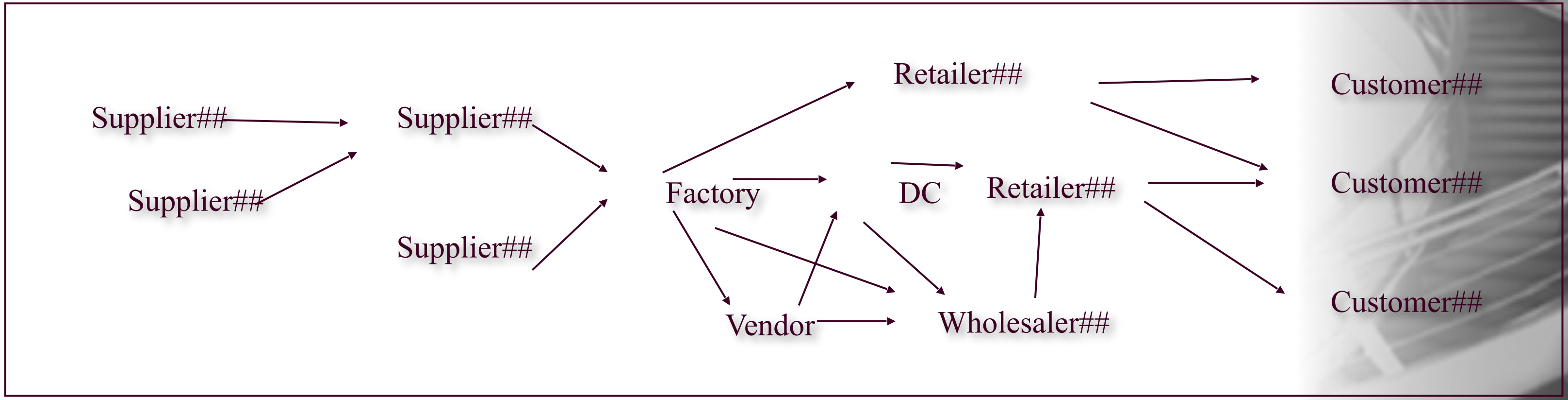
- Each of the problems of this chapter can be formulated as linear programs and solved by general purpose linear programming codes.
- For each of the problems, if the right-hand side of the linear programming formulations are all integers, the optimal solution will be in terms of integer values for the decision variables.
- However, there are many computer packages that contain separate computer codes for these problems which take advantage of their network structure.

Supply Chain Models

- A <u>supply chain</u> describes the set of all interconnected resources involved in producing and distributing a product.
- In general, supply chains are designed to satisfy customer demand for a product at minimum cost.
- Those that control the supply chain must make decisions such as where to produce a product, how much should be produced, and where it should be sent.

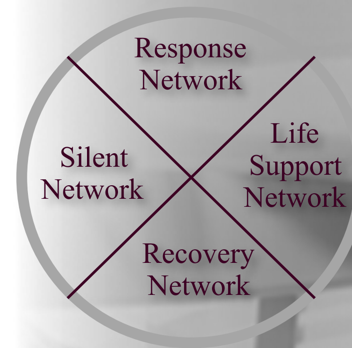
Supply Chain Management

- SC = a sales-production-distribution network
- M = enabling conditions and enhancing (contractual, goodwill, competence) trust for supply chain excellence



Supply Chain Operations

Trigger::Acute Shocks (e.g. natural hazards)



Long Term **Forecasting**	**Options of Network Topology (Decision and/or Game Theory)**	
Medium Term **Forecasting** (1 month – 1 yea)	**Network Topology Fixed (Project Management)**	⟺ **Inventory Module** / **Simulation Module**
Short Term **Forecasting** (1-4 weeks)	**Network Topology and Fleet Fixed (Transportation Method)**	⟺ **Linear Programming Module**

WAL-MART'S SUPPLY CHAIN *MANAGEMENT* PRACTICES ..... faster *inventory* turnover, accurate *forecasting* of *inventory* levels, increased warehouse space,

- http://mohanchandran.files.wordpress.com/2008/01/wal-mart.pdf

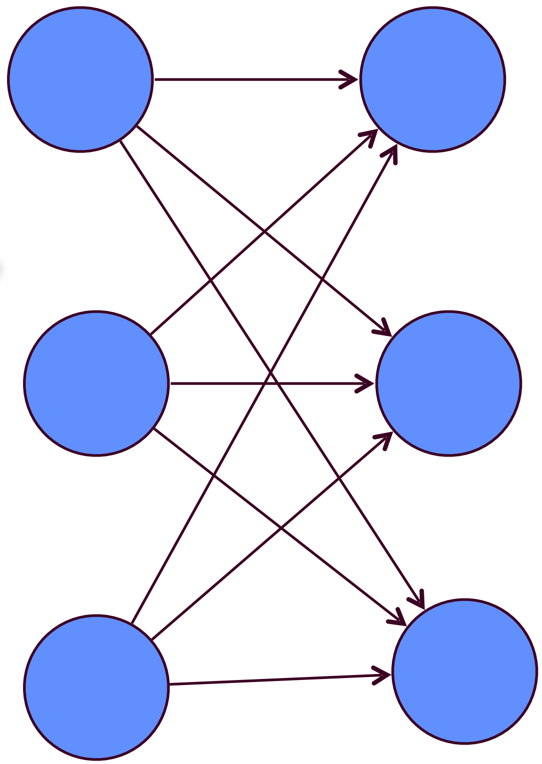
I. Guidelines for Transportation Model Formulation and Solution Strategies

- Define the decision variables
- Write the objective in terms of the decision variables.
- Write the constraints in terms of the decision variables.
- Construct a spreadsheet model and solve for the optimal solution using Excel Solver.

I. Transportation Problem

Supply or origins
(i)

Demand or destinations
(j)



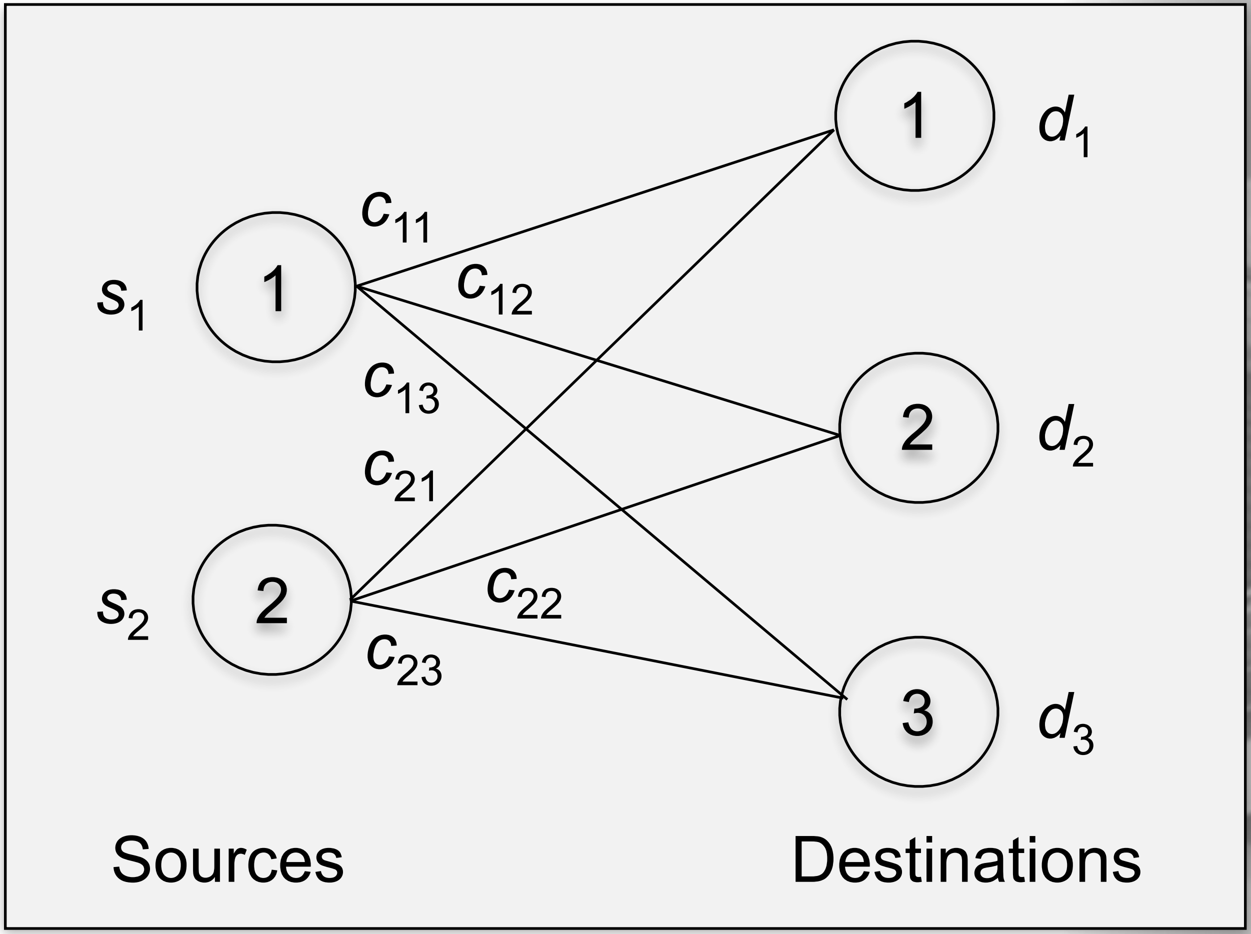
How many decision variables would there be?

Xij: No. of goods shipped from origin i to destination j

Transportation Problem

- The transportation problem seeks to minimize the total shipping costs of transporting goods from m origins (each with a supply s_i) to n destinations (each with a demand d_j), when the unit shipping cost from an origin, i, to a destination, j, is c_{ij}.
- The network representation for a transportation problem with two sources and three destinations is given on the next slide.

- Network Representation



Transportation Problem

- Linear Programming Formulation

Using the notation:

x_{ij} = number of units shipped from origin *i* to destination *j*

c_{ij} = cost per unit of shipping from origin *i* to destination *j*

s_i = supply or capacity in units at origin *i*

d_j = demand in units at destination *j*

continued →

Transportation Problem

- Linear Programming Formulation (continued)

$$\text{Min} \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} \le s_i \quad for\ i = 1, 2, \ldots, m \quad \text{Supply}$$

$$\sum_{i=1}^{m} x_{ij} = d_j$$ for ~~i~~ j $= 1, 2, \ldots, n$ Demand

$x_{ij} \geq 0$ for all *i* and *j*

Using Excel Solver

1. Develop your Excel model
   - Parameters
   - Decision Variables
   - Model Outputs

=SUMPRODUCT(array 1, array 2)

=SUM(a range of cells)

the **Sum** function adds all numbers in a range of cells and returns the result

2. Use the Solver to find solutions

Transportation Problem

- LP Formulation Special Cases
  - Total supply exceeds total demand:

    No modification of LP formulation is necessary.

  - Total demand exceeds total supply:

    Add a dummy origin with supply equal to the shortage amount. Assign a zero shipping cost per unit. The amount "shipped" from the dummy origin (in the solution) will not actually be shipped.

Transportation Problem

- LP Formulation Special Cases (continued)
  - The objective is maximizing profit or revenue:

    Solve as a maximization problem.

  - Minimum shipping guarantee from *i* to *j*:

    $x_{ij} \geq L_{ij}$

  - Maximum route capacity from *i* to *j*:

    $x_{ij} \leq L_{ij}$

  - Unacceptable route:

    Remove the corresponding decision variable.

Transportation Problem: Example #1

Acme Block Company has orders for 80 tons of concrete blocks at three suburban locations as follows: Northwood -- 25 tons, Westwood -- 45 tons, and Eastwood -- 10 tons. Acme has two plants, each of which can produce 50 tons per week. Delivery cost per ton from each plant to each suburban location is shown on the next slide.

How should end of week shipments be made to fill the above orders?

Open the file (MGT355.Ch10.xls) and locate the worksheet (1.Example 1-Template).

Transportation Problem: Example #1

- Delivery Cost Per Ton

	Northwood	Westwood	Eastwood
Plant 1	24	30	40
Plant 2	30	40	42

Transportation Problem: Example #1

- Optimal Solution

Variable	From	To	Amount	Cost
x_{11}	Plant 1	Northwood	5	120
x_{12}	Plant 1	Westwood	45	1,350
x_{13}	Plant 1	Eastwood	0	0
x_{21}	Plant 2	Northwood	20	600
x_{22}	Plant 2	Westwood	0	0
x_{23}	Plant 2	Eastwood	10	420
			Total Cost =	$2,490

Transportation Problem: Example #1 – What IF

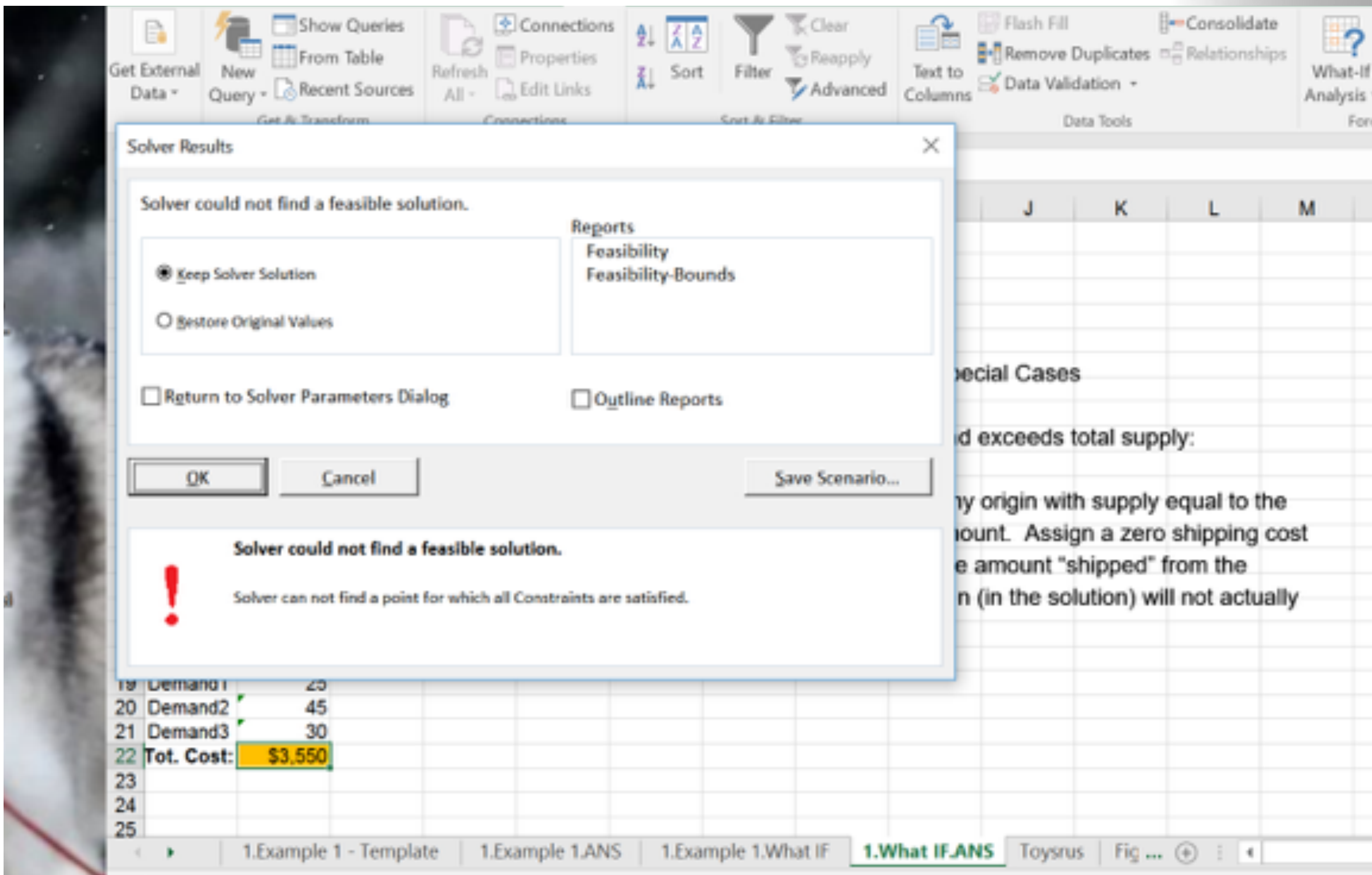
LP Formulation Special Cases

- Total demand exceeds total supply: Eastwood -- **40** tons

  Add a dummy origin (i.e. Plant 3) with supply equal to the shortage amount. Assign a zero shipping cost per unit. The amount "shipped" from the dummy origin (in the solution) will not actually be shipped.

Open the file (MGT355.Ch10.xls) and locate the worksheet (1.Example 1-What IF).

Transportation Problem: Example #1 – What IF



Transportation Problem: Example #2

FIGURE 10.1 THE NETWORK REPRESENTATION OF THE FOSTER GENERATORS TRANSPORTATION PROBLEM

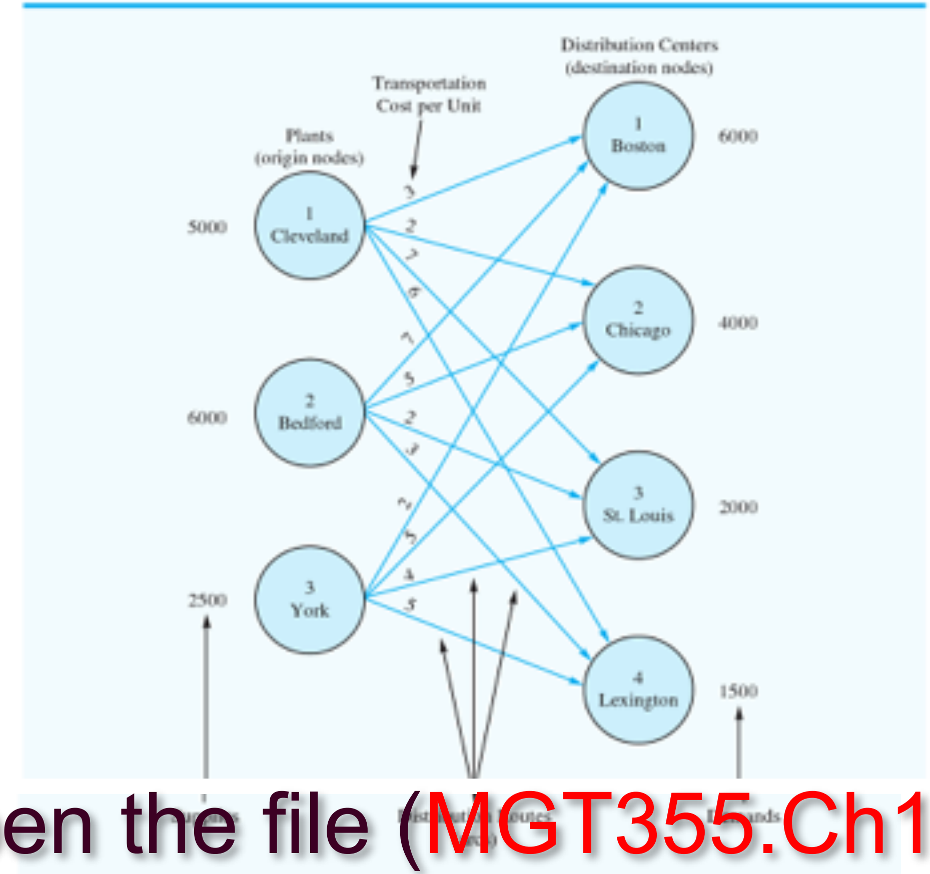


Image Credit: Text book

Open the file (MGT355.Ch10.xls) and locate the worksheet (2.Figure10.1).

Transportation Problem: Example #2

Image Credit: Text book

$$\text{Min} \quad 3x_{11} + 2x_{12} + 7x_{13} + 6x_{14} + 7x_{21} + 5x_{22} + 2x_{23} + 3x_{24} + 2x_{31} + 5x_{32} + 4x_{33} + 5x_{34}$$

s.t.

$$
\begin{aligned}
x_{11} + x_{12} + x_{13} + x_{14} &\leq 5000 \\
x_{21} + x_{22} + x_{23} + x_{24} &\leq 6000 \\
x_{31} + x_{32} + x_{33} + x_{34} &\leq 2500 \\
x_{11} + x_{21} + x_{31} &= 6000 \\
x_{12} + x_{22} + x_{32} &= 4000 \\
x_{13} + x_{23} + x_{33} &= 2000 \\
x_{14} + x_{24} + x_{34} &= 1500
\end{aligned}
$$

$x_{ij} \geq 0 \quad$ for $i = 1, 2, 3$ and $j = 1, 2, 3, 4$

Transportation Problem: Example #3

Downtown hospital has initiated a new procedure to ensure that patients receive their meals while the food is still as hot as possible. The hospital will continue to prepare the food in its kitchen but will now deliver it in bulk (not individual servings) to one of three new serving stations in the building. From there, the food will be reheated and meals will be placed on individual trays, loaded onto a cart, and distributed to the various floors and wings to the hospital.

Open the file (MGT355.Ch10.xls) and locate the worksheet (3.Hospital).

Transportation Problem: Example #3

The following table summarizes the number of trays that each station can serve.

Location	A	G	S
Capacity (Meals)	200	225	275

There are six wings that must be served. The following table reflects the number of patients in each wing.

Wing	1	2	3	4	5	6
Patients	80	120	150	210	60	80

Transportation Problem: Example #3

The purpose of the new procedure is to increase the temperature of the hot meals that the patient receives. Therefore, the amount of time needed to deliver a tray from a serving station will determine the proper distribution of food from serving station to wing. The following table summarizes the time (minutes) associated with each possible distribution channel.

Transportation Problem: Example #3

	Wing 1	Wing 2	Wing 3	Wing 4	Wing 5	Wing 6
Station A	12	11	8	9	6	6
Station G	6	12	7	7	5	8
Station S	8	9	6	6	7	9

What is your recommendation for handling the distribution of trays from the three serving stations? (Formulate an LP to achieve your goal.)

(Optional) What are the important elements of service quality? Explain how each contributes to the service quality? What analysis tools would you use?

Transportation Problem: Example #4

Ashley's Auto Top Carriers currently maintains plants in Atlanta and Tulsa that supply major distribution centers in Los Angeles and New York. Because of an expanding demand, Ashley has decided to open a third plant and has narrowed the choice to one of two cities—New Orleans or Houston. The pertinent production and distribution costs, as well as the plant capacities and distribution demands, are shown in the accompanying table. Which of the new possible plants should be opened?

Open the file (MGT355.Ch10.xls) and locate the worksheet (4.Location(option1)).

X_{ij}: No. of units shipped from plant i to center j

	To Distribution Centers From Plants	Los Angeles	New York	Normal Production	Unit Production Cost ($)
Existing Plants	Atlanta	$8	$5	600	6
	Tulsa	$4	$7	900	5
Proposed Locations	New Orleans	$5	$6	500	4 (anticipated)
	Houston	$4	$6*	500	3 (anticipated)
	Forecast Demand	800	1,200	2,000	

* Indicates distribution cost (shipping, handling, storage) will be $6 per carrier if sent from Houston to New York

System with New Orleans Factory

	Los Angeles	New York	
Atlanta			600
Tulsa			900
New Orleans			**500**
	800	1200	

System with Houston Factory

	Los Angeles	New York	
Atlanta			600
Tulsa			900
Houston			**500**
	800	1200	

Transportation Problem: Example #5

The Navy has 9,000 pounds of material in Albany, Georgia that it wishes to ship to three installations: San Diego, Norfolk, and Pensacola. They require 4,000, 2,500, and 2,500 pounds, respectively. Government regulations require equal distribution of shipping among the three carriers.

Open the file (MGT355.Ch10.xls) and locate the worksheet (5.Example 5).

Transportation Problem: Example #5

The shipping costs per pound for truck, railroad, and airplane transit are shown on the next slide. Formulate and solve a linear program to determine the shipping arrangements (mode, destination, and quantity) that will minimize the total shipping cost.

Transportation Problem: Example #5

	Destination		
Mode	San Diego	Norfolk	Pensacola
Truck	$12	$ 6	$ 5
Railroad	20	11	9
Airplane	30	26	28

Transportation Problem: Example #5

- Define the Decision Variables

We want to determine the pounds of material, x_{ij} , to be shipped by mode *i* to destination *j*. The following table summarizes the decision variables:

	San Diego	Norfolk	Pensacola
Truck	x_{11}	x_{12}	x_{13}
Railroad	x_{21}	x_{22}	x_{23}
Airplane	x_{31}	x_{32}	x_{33}

- Define the Objective Function

Minimize the total shipping cost.

Min: (shipping cost per pound for each mode per destination pairing) x (number of pounds shipped by mode per destination pairing).

$$\text{Min } 12x_{11} + 6x_{12} + 5x_{13} + 20x_{21} + 11x_{22} + 9x_{23} + 30x_{31} + 26x_{32} + 28x_{33}$$

Transportation Problem: Example #5

- Define the Constraints

Equal use of transportation modes:

(1) $x_{11} + x_{12} + x_{13} = 3000$

(2) $x_{21} + x_{22} + x_{23} = 3000$

(3) $x_{31} + x_{32} + x_{33} = 3000$

Destination material requirements:

(4) $x_{11} + x_{21} + x_{31} = 4000$

(5) $x_{12} + x_{22} + x_{32} = 2500$

(6) $x_{13} + x_{23} + x_{33} = 2500$

Non-negativity of variables: $x_{ij} \geq 0$, i and $j = 1, 2, 3$

Transportation Problem: Example #5

Computer Output

Objective Function Value = 142000.000

Variable	Value
x_{11}	1000.000
x_{12}	2000.000
x_{13}	0.000
x_{21}	0.000
x_{22}	500.000
x_{23}	2500.000
x_{31}	3000.000
x_{32}	0.000
x_{33}	0.000

Transportation Problem: Example #5

- Solution Summary
  - San Diego will receive 1000 lbs. by truck and 3000 lbs. by airplane.
  - Norfolk will receive 2000 lbs. by truck and 500 lbs. by railroad.
  - Pensacola will receive 2500 lbs. by railroad.
  - The total shipping cost will be $142,000.

II. Transshipment Problem

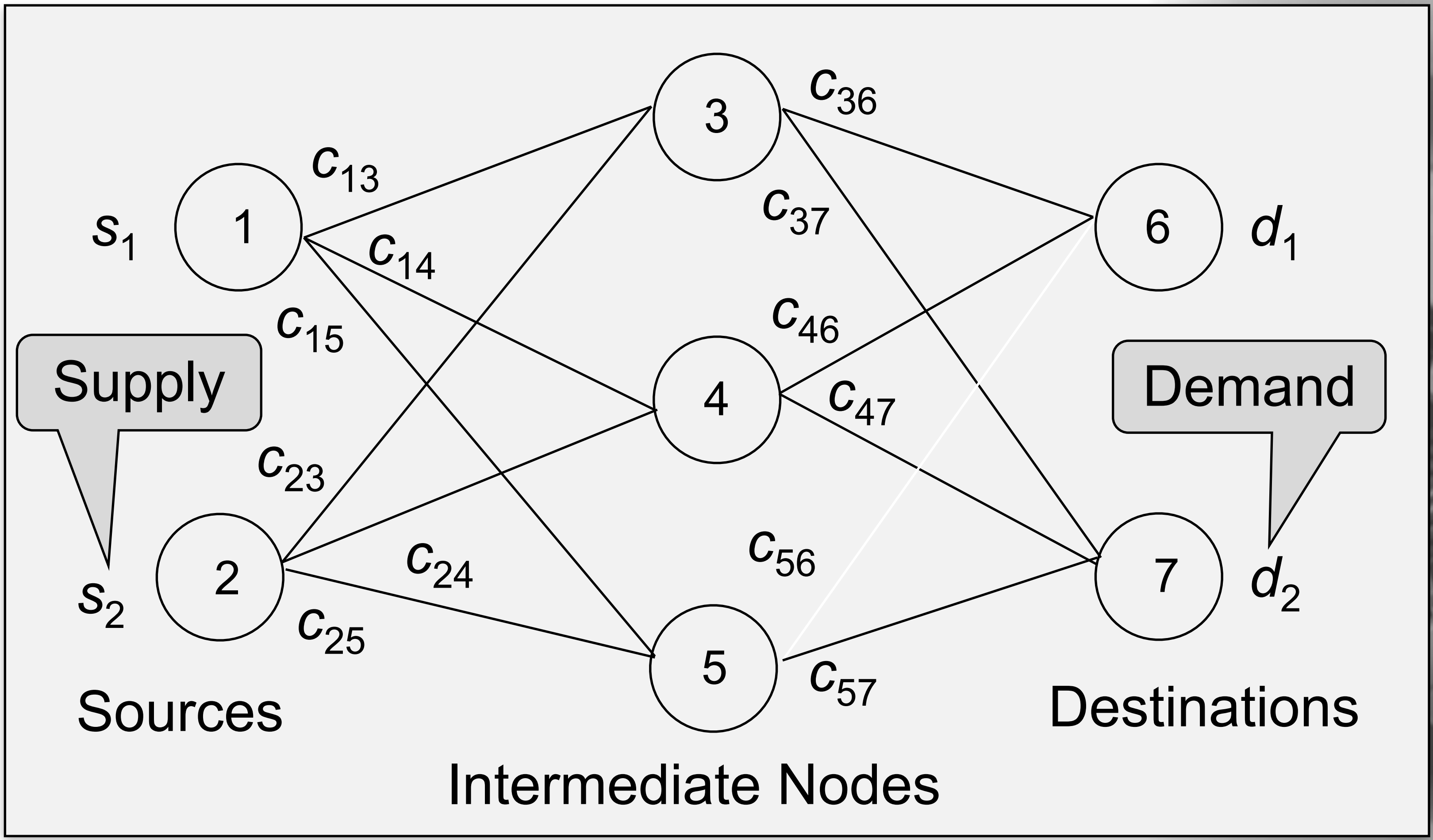
- transshipment problem: in practice, some locations are used as intermediate shipping points (known as transshipment points). They serve both as origins and as destinations.
- Farm Market Problem
- Figure 10.4
- Figure 10.7
- Practice exam (1) – Problem 8
- Zeron Industries

II. Transshipment Problem

- Transshipment problems are transportation problems in which a shipment may move through intermediate nodes (transshipment nodes) before reaching a particular destination node.
- Transshipment problems can be converted to larger transportation problems and solved by a special transportation program.
- Transshipment problems can also be solved by general purpose linear programming codes.
- The network representation for a transshipment problem with two sources, three intermediate nodes, and two destinations is shown on the next slide.

II. Transshipment Problem

- Network Representation



II. Transshipment Problem

- Linear Programming Formulation

Using the notation:

x_{ij} = number of units shipped from node *i* to node *j*

c_{ij} = cost per unit of shipping from node *i* to node *j*

s_i = supply at origin node *i*

d_j = demand at destination node *j*

continued →

II. Transshipment Problem

- LP Formulation Special Cases
  - Total supply not equal to total demand
  - Maximization objective function
  - Route capacities or route minimums
  - Unacceptable routes

  The LP model modifications required here are identical to those required for the special cases in the transportation problem.

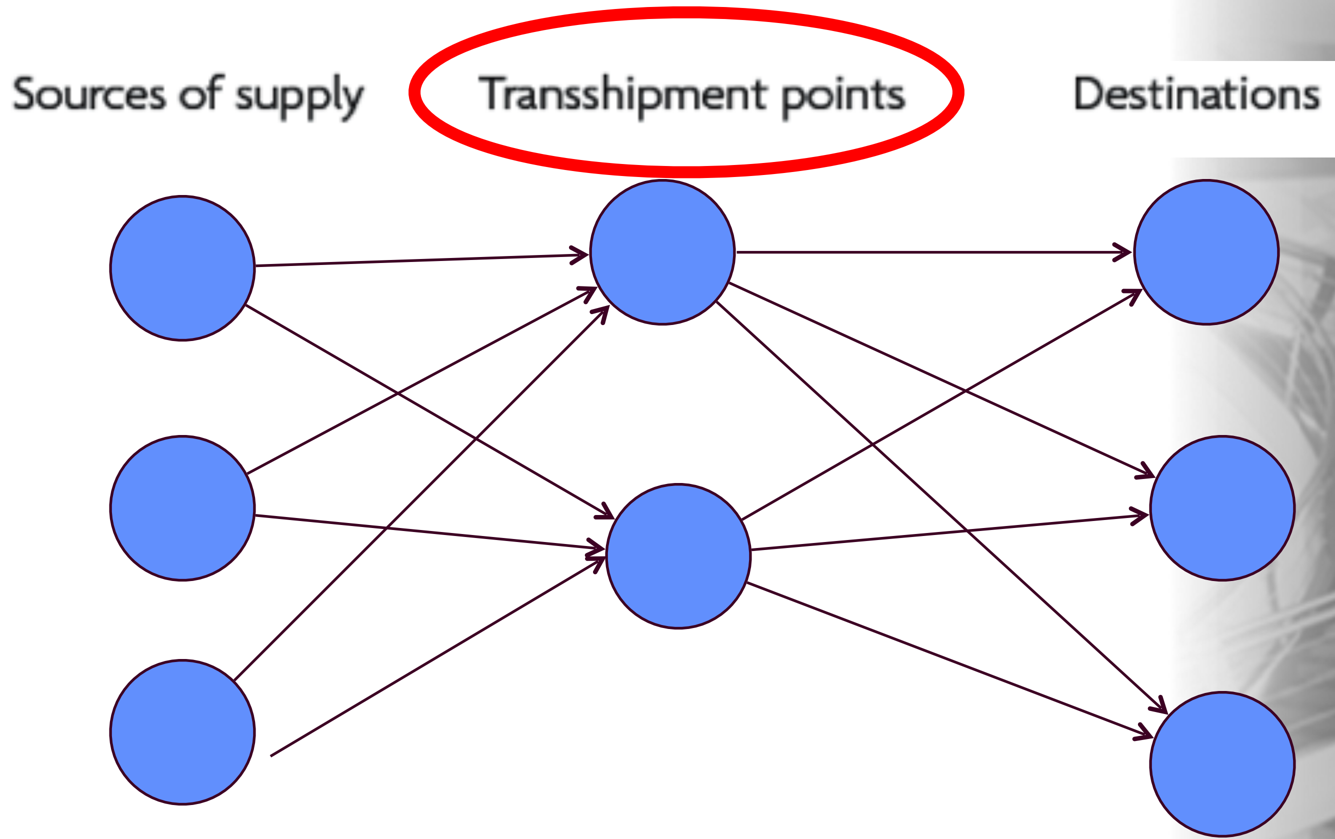
Example in Point 1

Farm Market Problem

		Market	Demand (units)
Farm	Capacity (units)	I	50
A	100	II	150
B	200	III	300
C	200		

Open the file (MGT355.Ch10.xls) and locate the worksheet (6.Farm-Market).

The manager of Farm-Market has decided to utilize two intermediate nodes as transshipment points for temporary storage of goods. The revised diagram of the transshipment problem is given below::



Xij: No. of goods shipped from origin i to destination j

- The unit transportation costs (in $) for shipments from the three farms to the two warehouses and from the two warehouses to the three markets are as follows:

	Warehouse	
Farm	ONE	TWO
A	**3**	**2**
B	**4**	**3**
C	**2.5**	**3.5**

	Market		
Warehouse	I	II	III
ONE	**2**	**1**	**4**
TWO	**3**	**2**	**5**

Solve the linear program to determine the minimum cost shipping schedule for the problem.

Example in Point 2 **Figure 10.4**

FIGURE 10.4 NETWORK REPRESENTATION OF THE RYAN ELECTRONICS TRANSSHIPMENT PROBLEM

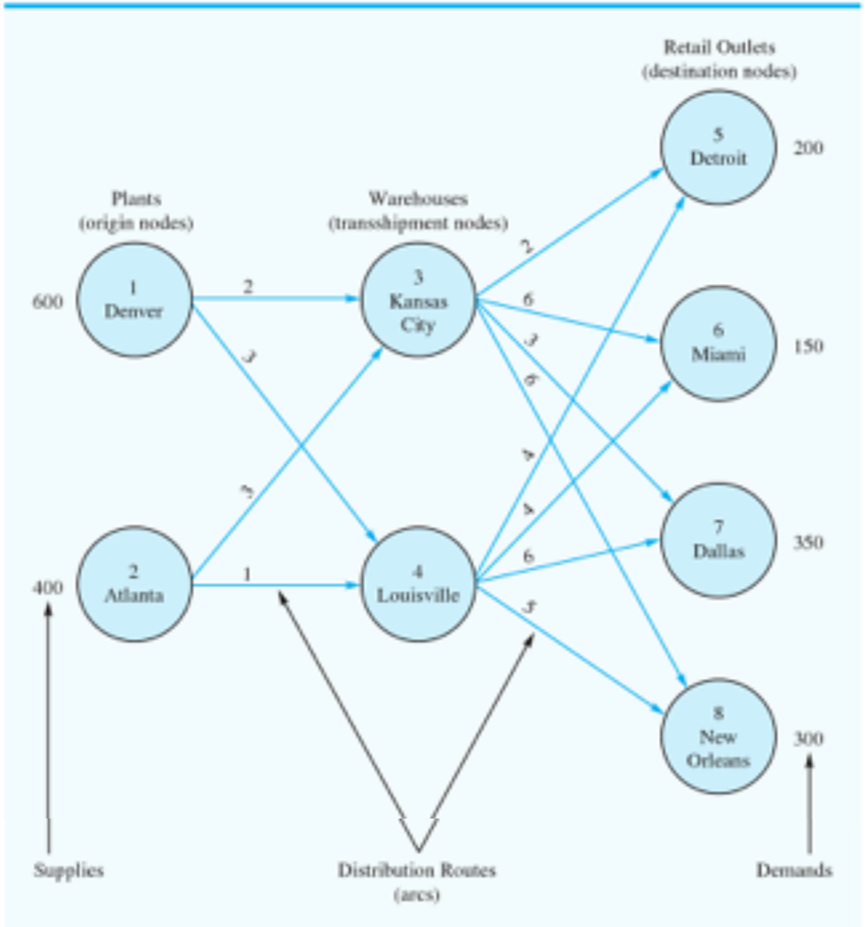


Image Credit:
Text book

Example in Point 2

Image Credit: Text book

FIGURE 10.5 LINEAR PROGRAMMING FORMULATION OF THE RYAN ELECTRONICS TRANSSHIPMENT PROBLEM

$$\text{Min } 2x_{13} + 3x_{14} + 3x_{23} + 1x_{24} + 2x_{35} + 6x_{36} + 3x_{37} + 6x_{38} + 4x_{45} + 4x_{46} + 6x_{47} + 5x_{48}$$

s.t.

$$\left.\begin{aligned} x_{13} + x_{14} &\le 600 \\ x_{23} + x_{24} &\le 400 \end{aligned}\right\} \text{Origin node constraints}$$

$$\left.\begin{aligned} -x_{13} - x_{23} + x_{35} + x_{36} + x_{37} + x_{38} &= 0 \\ -x_{14} - x_{24} + x_{45} + x_{46} + x_{47} + x_{48} &= 0 \end{aligned}\right\} \text{Transshipment node constraints}$$

$$\left.\begin{aligned} x_{35} + x_{45} &= 200 \\ x_{36} + x_{46} &= 150 \\ x_{37} + x_{47} &= 350 \\ x_{38} + x_{48} &= 300 \end{aligned}\right\} \text{Destination node constraints}$$

$x_{ij} \ge 0$ for all i and j

Open the file (MGT355.Ch10.xls) and locate the worksheet (7.Figure 10.4).

Example in Point 3

Figure 10.7

FIGURE 10.7 NETWORK REPRESENTATION OF THE MODIFIED RYAN ELECTRONICS TRANSSHIPMENT PROBLEM

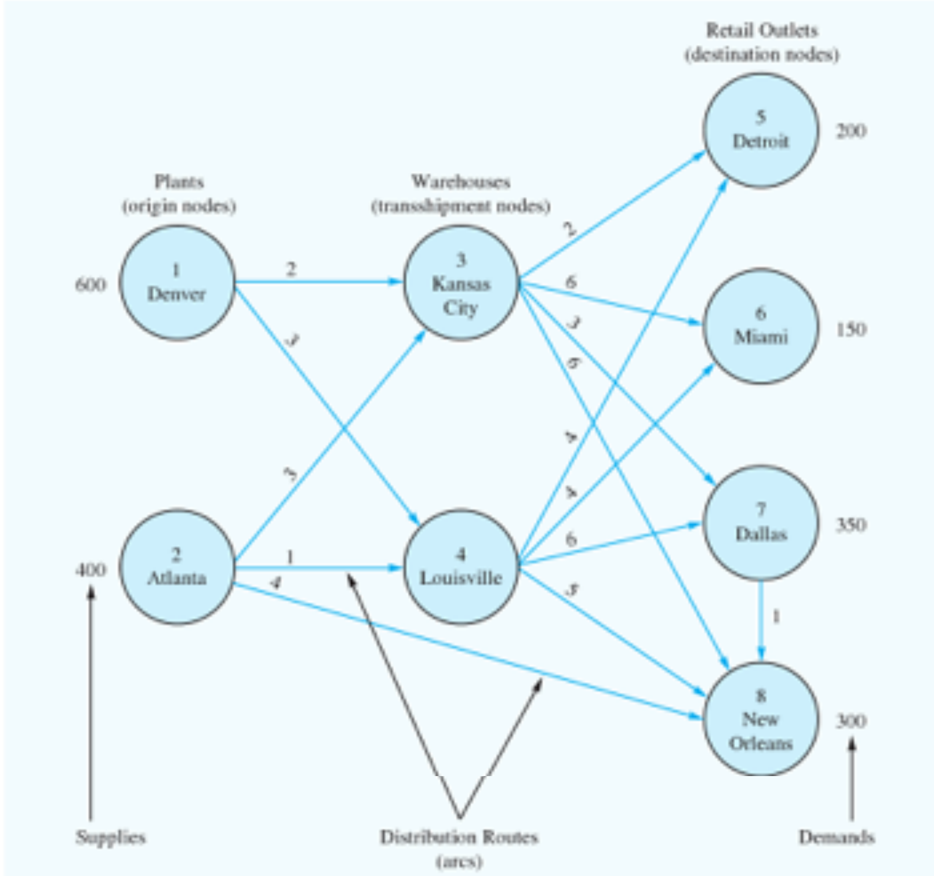


Image Credit:
Text book

Example in Point 3

Image Credit:
Text book

FIGURE 10.8 LINEAR PROGRAMMING FORMULATION OF THE MODIFIED RYAN ELECTRONICS TRANSSHIPMENT PROBLEM

$$\text{Min} \quad 2x_{13} + 3x_{14} + 3x_{23} + 1x_{24} + 2x_{35} + 6x_{36} + 3x_{37} + 6x_{38} + 4x_{45} + 4x_{46} + 6x_{47} + 5x_{48} + 4x_{28} + 1x_{78}$$

s.t.

$$x_{13} + x_{14} \leq 600$$
$$x_{23} + x_{24} + x_{28} \leq 400$$

Origin node constraints

$$-x_{13} - x_{23} + x_{35} + x_{36} + x_{37} + x_{38} = 0$$
$$-x_{14} - x_{24} + x_{45} + x_{46} + x_{47} + x_{48} = 0$$

Transshipment node constraints

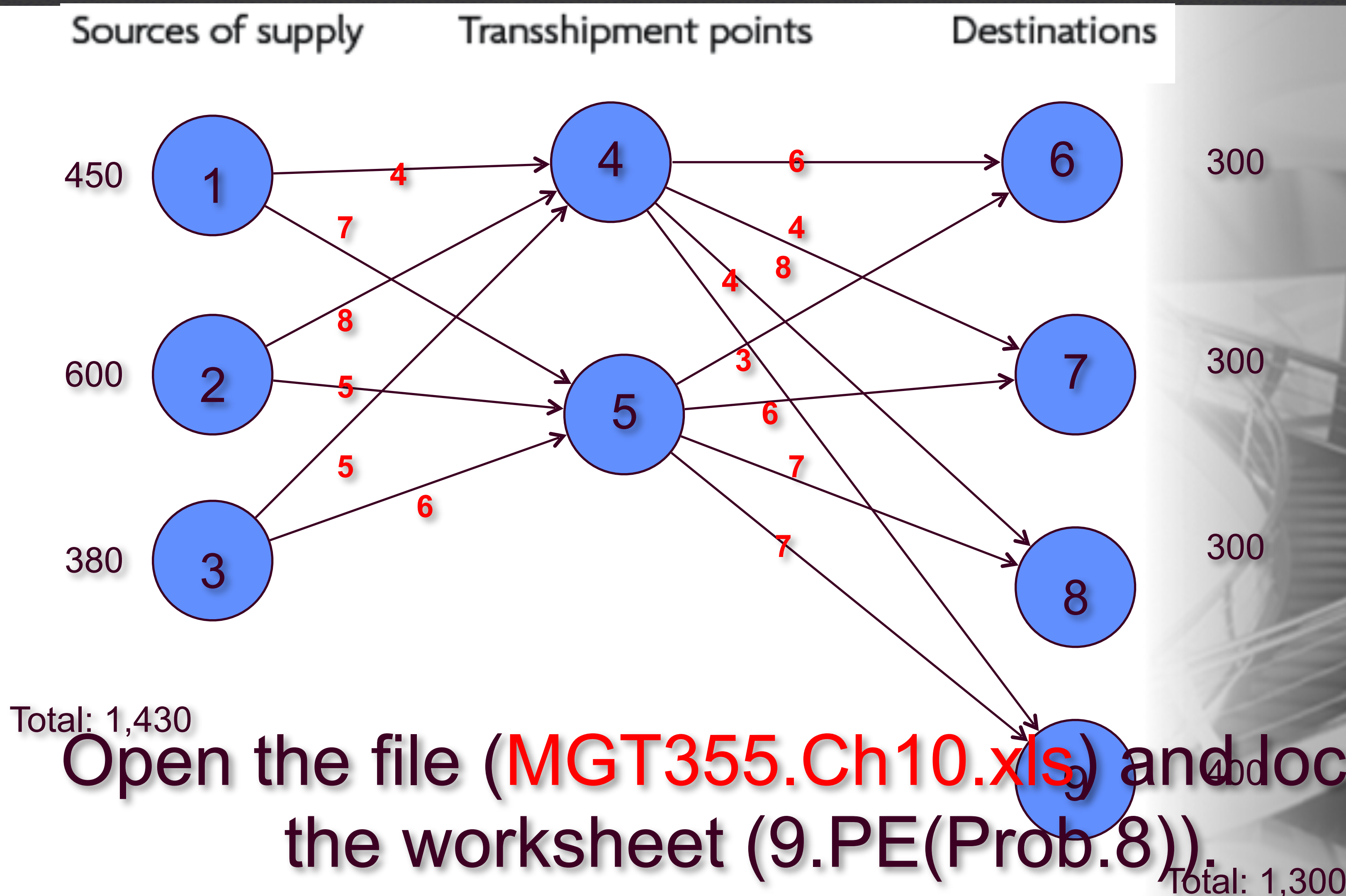
$$x_{35} + x_{45} = 200$$
$$x_{36} + x_{46} = 150$$
$$x_{37} + x_{47} - x_{78} = 350$$
$$x_{38} + x_{48} + x_{28} + x_{78} = 300$$

Destination node constraints

$$x_{ij} \geq 0 \quad \text{for all } i \text{ and } j$$

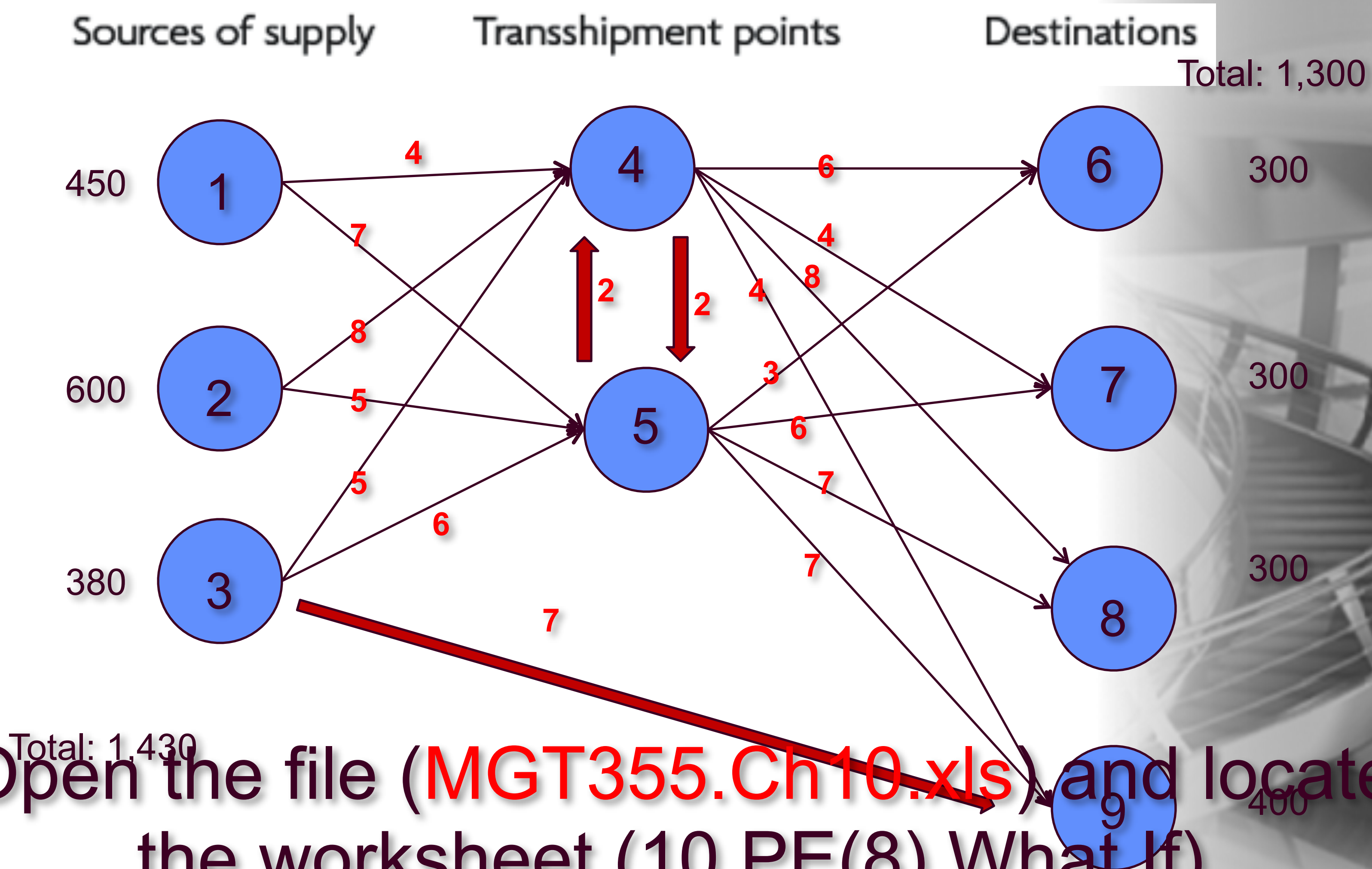
Open the file (MGT355.Ch10.xls) and locate the worksheet (8.Figure 10.7).

Example in Point 4 Practice Exam (1, pages 24-25)



Open the file (MGT355.Ch10.xls) and locate the worksheet (9.PE(Prob.8)).

Example in Point 5 Practice Exam (1, pages 25-26)



Open the file (MGT355.Ch10.xls) and locate the worksheet (10.PE(8) What If).

Example in Point 6 — Transshipment Problem

The Northside and Southside facilities of Zeron Industries supply three firms (Zrox, Hewes, Rockrite) with customized shelving for its offices. They both order shelving from the same two manufacturers, Arnold Manufacturers and Supershelf, Inc.

Currently weekly demands by the users are 50 for Zrox, 60 for Hewes, and 40 for Rockrite. Both Arnold and Supershelf can supply at most 75 units to its customers.

Additional data is shown on the next slide.

Open the file (MGT355.Ch10.xls) and locate the worksheet (11.DIY).

Example in Point 6

Because of long standing contracts based on past orders, unit costs from the manufacturers to the suppliers are:

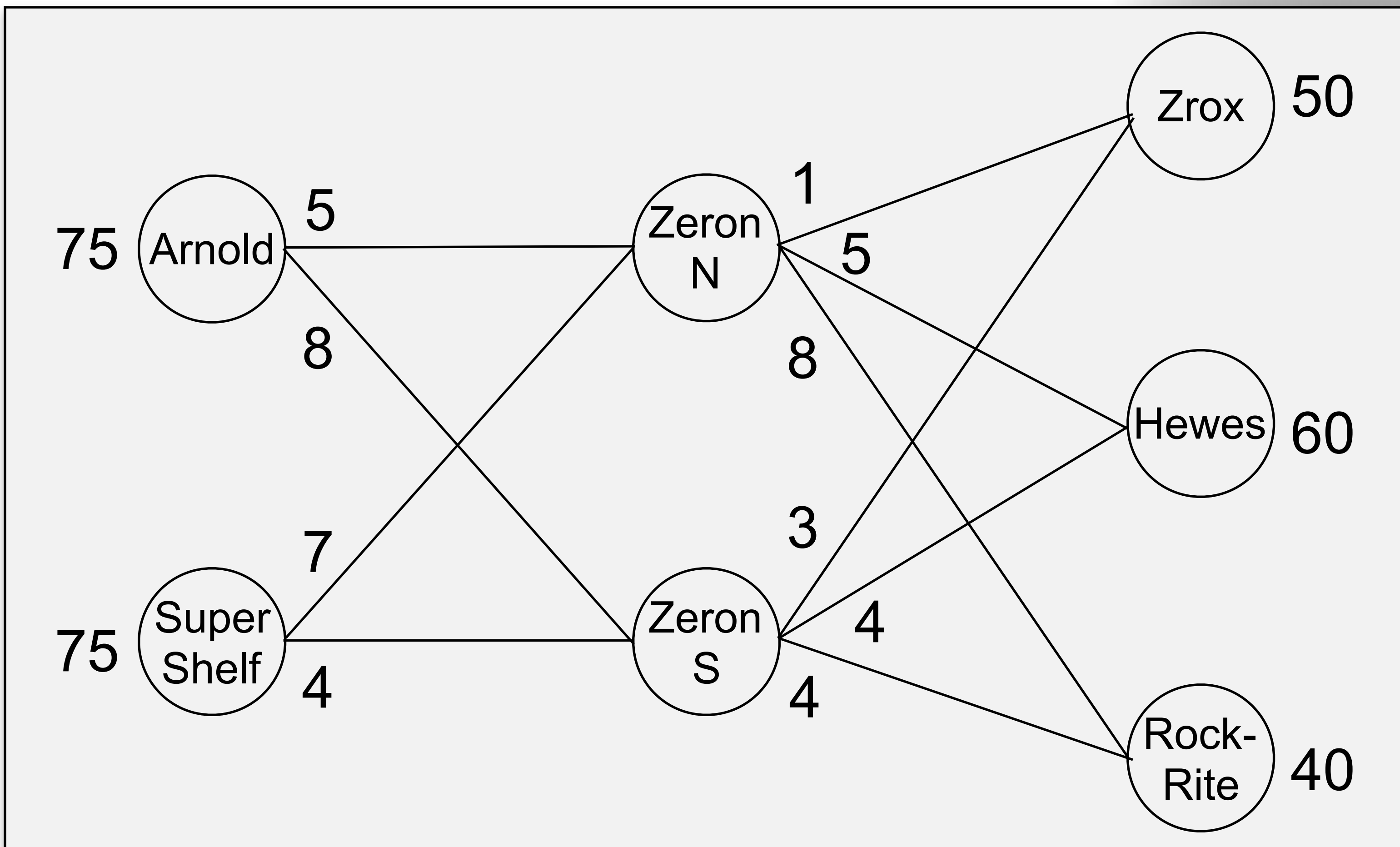
	Zeron N	Zeron S
Arnold	5	8
Supershelf	7	4

The costs to install the shelving at the various locations are:

	Zrox	Hewes	Rockrite
Thomas	1	5	8
Washburn	3	4	4

Example in Point 6

- Network Representation



Example in Point 6

- Linear Programming Formulation
  - Decision Variables Defined

x_{ij} = amount shipped from manufacturer i to supplier j

x_{jk} = amount shipped from supplier j to customer k

where i = 1 (Arnold), 2 (Supershelf)

j = 3 (Zeron N), 4 (Zeron S)

k = 5 (Zrox), 6 (Hewes), 7 (Rockrite)

Example in Point 6

- Linear Programming Formulation
  - Objective Function Defined

    Minimize Overall Shipping Costs:

$$\text{Min} \quad 5x_{13} + 8x_{14} + 7x_{23} + 4x_{24} + 1x_{35} + 5x_{36} + 8x_{37} + 3x_{45} + 4x_{46} + 4x_{47}$$

Example in Point 6

- Constraints Defined

Amount Out of Arnold: $x_{13} + x_{14} \leq 75$

Amount Out of Supershelf: $x_{23} + x_{24} \leq 75$

Amount Through Zeron N: $x_{13} + x_{23} - x_{35} - x_{36} - x_{37} = 0$

Amount Through Zeron S: $x_{14} + x_{24} - x_{45} - x_{46} - x_{47} = 0$

Amount Into Zrox: $x_{35} + x_{45} = 50$

Amount Into Hewes: $x_{36} + x_{46} = 60$

Amount Into Rockrite: $x_{37} + x_{47} = 40$

Non-negativity of Variables: $x_{ij} \geq 0$, for all *i* and *j*

Example in Point 6

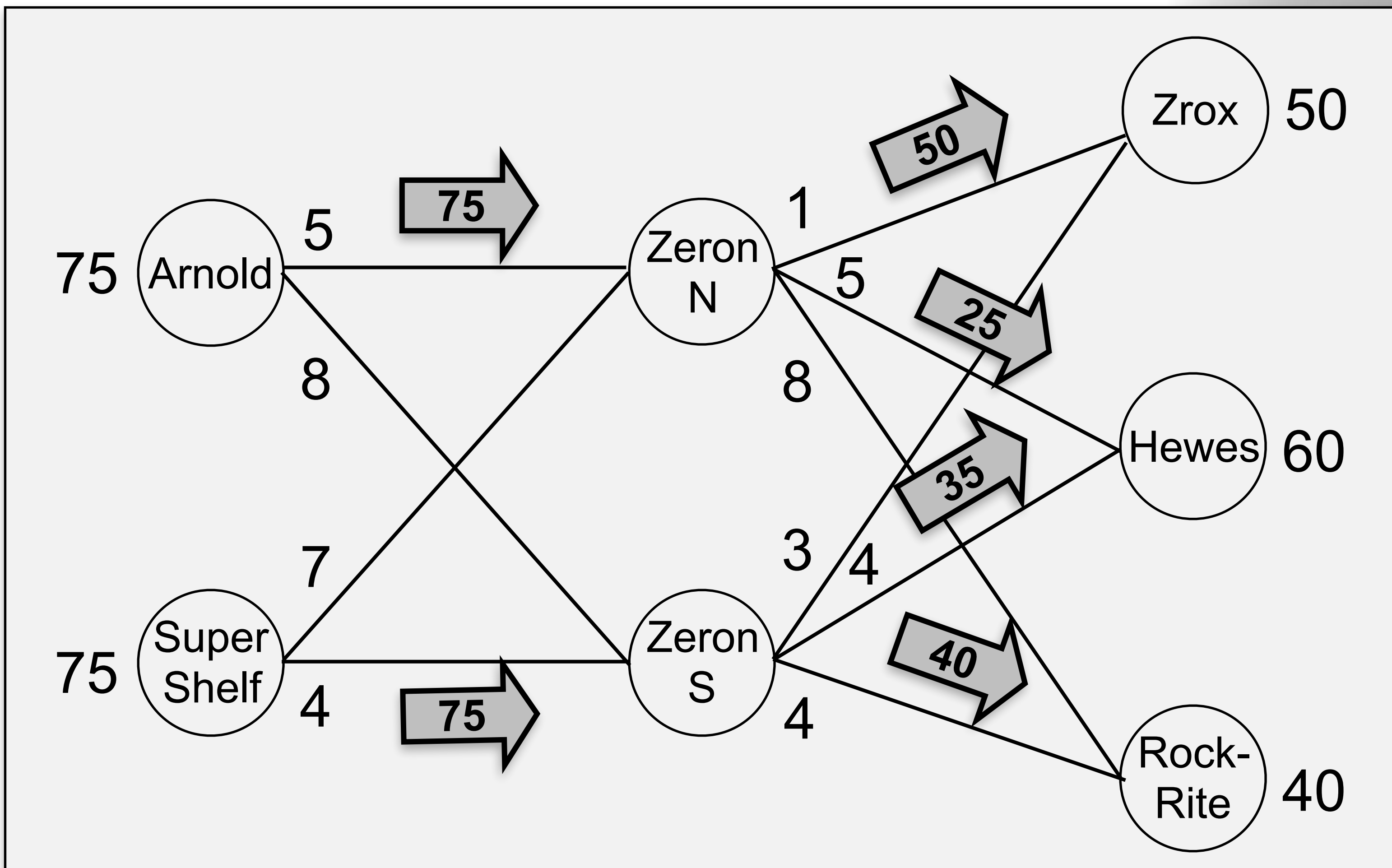
- Computer Output

Objective Function Value = 1150.000

Variable	Value
X13	75.000
X14	0.000
X23	0.000
X24	75.000
X35	50.000
X36	25.000
X37	0.000
X45	0.000
X46	35.000
X47	40.000

Example in Point 6

- Solution



I-assignment

Examples of Optimization Problems - Frontline Systems

http://www.solver.com/optimization-examples.htm

Focus your attention on the following:

Transportation Problems